TR-10

MODEL ROCKET
TECHNICAL REPORT

ALTITUDE PREDICTION CHARTS

ESTES®

TR-10

Model Rocket Altitude Prediction Charts

©Copyright 1967, 1970, 1971, 1999 Centuri Corporation.

All rights reserved.

EST 2842

PREFACE

This report presents a relatively simple method by which aerodynamic drag effects can accurately be taken into account in the prediction of model rocket peak altitudes. With this data, altitudes can be determined for any rocket using any of the Estes motors (including 2 or 3 stage vehicles and cluster-powered rockets). In addition, flight times can be easily found so that optimum engine delay times can be selected.

Using this simple method for calculating aerodynamic drag effects, many interesting experiments and research projects can be initiated which would have previously been too laborious for easy analysis. Examination of the easy-to-read graphs and a few simple arithmetic calculations will enable you to accurately predict the performances of your model rockets with different Estes engines. Also, a basic understanding of the principles of aerodynamics and the aerospace sciences can be obtained by performing the simple drag experiments suggested at the end of this report.

TABLE OF CONTENTS

INTRODUCTION

When rocketeers get together, the subject for discussion is often “How high do our rockets go?” A good solution to this problem has many uses in rocket design from building the wildest sporter to perfecting the sleekest altitude model.

This report presents the technique of conveniently calculating all the pertinent parameters you need: burnout altitude and velocity, peak altitude and coast time to peak. The technique is useful for all types of models including clustered rockets, staged rockets and boostgliders.

For example, with the information you calculate you can pick the optimum weights for the engines you use, the best engine for your model weight and the correct delay time for your engine and model combination. You can investigate the performance change due to variation of parameters such as weight, diameter and drag. Further investigation can tell you if the weight of a modification will be offset by reduced drag or vice-versa.

A complete “parameter trade-off study” could be a prize winning science fair project and should be part of any high performance rocket design.

DISCUSSION OF DRAG

THEORETICAL "NO DRAG" CONDITIONS VERSUS ACTUAL FLIGHT CONDITIONS

In the June 1964, "Rocket Math" article (reference I, page 36) it was pointed out that "typically, drag will reduce the peak altitude of a shortened Astron Streak* with a 1/2 A8-4* engine aboard from a theoretical drag free 1935 feet to an actual 710 feet". What this means is simply that merely due to the effect of air the altitude has been cut to less than half. The big questions is, "Why is this effect so great and what is happening to the rocket to cause such a difference?"

To better understand the effect of the atmospheric forces that retard the forward motion of the rocket (We are talking about "drag" again) let us consider the following example. Suppose we have a one inch diameter rocket (approximate size of BT-50) traveling at 100 ft./sec. We can determine the number of air molecules which collide with our rocket each second by using Loschmidts number, which is the number of molecules in a cubic centimeter of gas under "standard atmosphere" conditions (temperature of 32˚ F or 0˚ C) and pressure of 14.7 pounds per square inch, or 76 cm of mercury or 10.1 newton-seconds /cm).

It turns out that during each second of flight time our 100 ft./sec. rocket has to push its way through approximately 400,000,000,000,000,000,000,000 molecules that lie in its path. (Scientists and engineers usually write out such large numbers in the form 4×10^{23}, which means 4 followed by 23 zeros).

Even though molecules are very small, this extremely large number of them can add up to a drag force of about 1 ounce acting continuously at a speed of 100 ft./sec. Since drag is proportional to the square of the velocity,** we would then have 4 ounces of drag at 200 ft./sec. and 9 ounces of drag on the rocket at 300 ft./sec. (approximately 200 miles per hour).

Fortunately, the full effect of these molecules sitting in the path of our rocket can be avoided by good streamlining. In this way, most of the molecules are compressed in layers that tend to flow smoothly around the nose and body tube as the rocket passes by them. Relatively few molecules are really rammed head-on.

*This rocket or engine is no longer available.

**The square of a number is obtained by multiplying the number times itself.

These millions of impacts constitute the effects more commonly known as "aerodynamic drag" on a rocket or, more simply, as just "drag". Drag on any object has been found to follow this law:

$$D = C_D A \tfrac{1}{2} \rho V^2$$

- **D** is the drag force.
- **C_D** is a dimensionless "aerodynamic drag coefficient" that depends upon the shape and the surface smoothness of the object.
- **A** is the reference area of the object (for model rock ets we use the cross sectional area of the body tube as the reference)
- ρ (pronounced "row") is the density of the medium through which the object is moving (submarine designers use the density of water in their drag computations, while model rocketeers use the density of air). The density symbol "ρ" is the Greek letter RHO.
- V is the velocity of the object

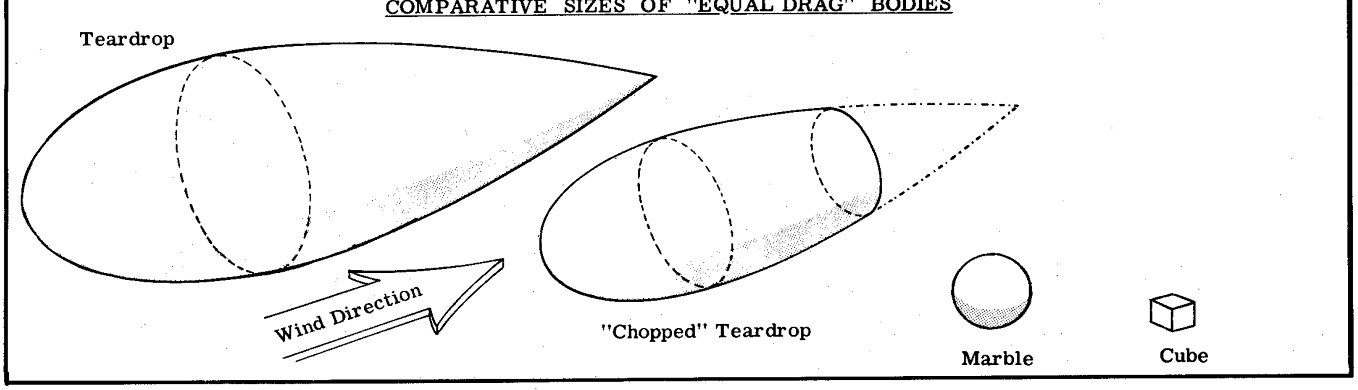
The full importance of drag has yet to be realized by most model rocketeers. It is hoped that through the use of the altitude prediction method presented herein that a more complete understanding of its effects will be acquired by all.

AERODYNAMIC DRAG COEFFICIENT

The **Aerodynamic Drag Coefficient** (C_D) is a measure of how easily a given shape moves "through" (passes by) the molecules of air.

For example, a cube traveling with a flat side forward has a C_D of 1.05. The following table gives you an idea of the importance of streamlining in reducing the **aerodynamic drag coefficient**.

Objects	C_D
Cube (flat side forward)	1.05
Sphere (as a marble)	.47
"Clipped" teardrop (streamlined teardrop shape with 1/3 of length removed at trailing edge)	.1
Streamlined teardrop shape	.05



COMPARATIVE SIZES OF "EQUAL DRAG" BODIES

DRAG FORM FACTOR

The product of the **aerodynamic drag coefficient** (C_D) and the frontal area of the object (A) is commonly referred to as the **Drag Form Factor** (C_DA) of the object.

Bodies with identical **drag form factor** will have identical drag value at any given speed.

For example, a one-inch cube moving with a flat side forward would have a **drag form factor** of 1.05 $inch^2$ (1.05 square inches). The data below provides additional examples to show the tremendous effect streamlining has on the drag of a body as reflected in the object's **drag form factor.** To provide easy comparisons, a **drag form factor** (C_DA) of 1.05 $inch^2$ has been selected.

Frontal Area of Object x Drag Coefficient = Drag Form Factor

A	X	C_D	=	C_DA
1 $inch^2$ cube	X	1.05	=	1.05 $inch^2$
2.23 $inch^2$ sphere	X	.47	=	1.05 $inch^2$
10.5 $inch^2$ clipped teardrop	X	.1	=	1.05 $inch^2$
21 $inch^2$ teardrop	X	.05	=	1.05 $inch^2$

This makes quite clear the great effect streamlining has on reducing drag. A streamlined teardrop with a frontal area of 21 square inches has the same **drag form factor** as a 1 inch cube.

The **drag form factor** for most models using standard Estes body tubes may be easily determined by consulting Figure 1, page 6. This graph was derived by calculating the cross-sectional area of each body tube and plotting this as a **Pressure Drag Coefficient** (C_D) of 1.0. Although the effective drag is not 100% of the cross-sectional area, the **drag form factor** for most body tubes at any selected **pressure drag coefficient** can be readily read from the graph or calculated should it occur beyond the limits of the graph.

"Aerodynamic Drag of Model Rocket," by G.M. Gregorek (Estes Technical Report TR-11), gives a much more complete discussion on drag than is contained in this report. Refer to TR-11 on drag to gain a more complete insight into the forces affecting drag on model rockets.

DISCUSSION OF THE BALLISTIC COEFFICIENT

Model rockets with high ballistic coefficients will reach higher altitudes than models with lower ballistic coefficients for a given weight and motor type.

There are several factors that affect the altitudes our rockets will reach. These factors are: the **Drag Coefficient** (C_D), the frontal area of the rocket (A), the rocket weight (W), the amount of propellant burned during thrusting (Wp), the type of motor used (D13 down to 1/4A) and even the temperature of the air and the altitude of the launch site.

The graphs in this report were made possible by combining several of these variables into one new variable call the Ballistic Coefficient β (Greek letter BETA).

$$\beta = \frac{W}{C_DA}$$

The **ballistic coefficient** is the ratio of the rocket weight divided by the **drag form factor** (C_DA), where the frontal area (A) is based on the body tube cross section area. The ballistic coefficient β is widely used by aerospace engineers as a trajectory parameter for space vehicles that are re-entering the atmosphere. It is also used by rifle designers who have to determine the best shape and weight of a bullet to use to obtain a given range and striking power.

A spacecraft with a high ballistic coefficient will come through the atmosphere faster than one with a low ballistic coefficient for a given set of initial re-entry conditions. A rifle bullet with a high ballistic coefficient will not slow down as fast as one with a lower ballistic coefficient.

SAMPLE PROBLEM

Assume that you have a model rocket that weighs 0.43 ounces without engines. An A8-3 engine is used to power the rocket. The rocket uses a BT-20 body tube. Assume an **aerodynamic drag coefficient** (C_D) of 0.75. This value is presented on page 94 of G.H. Stine's <u>Handbook of Model Rocketry.</u> Experiment 2 on Drag Coefficient Measurement will enable you to make your own decision on the validity of 0.75 for the C_D.

The numbers refer to the numbered steps in the instructions. (Refer to the instructions on page 3.)

1. Rocket empty weight = 0.43 oz.
   A8-3 engine weight = <u>0.57 oz.</u>
   Weight of rocket with engine = 1.00 oz.
   BT-20 tube: Diameter = .736 in.
   C_D=.75 (assumed)

2. Using a C_D of 0.75 and a BT-20 body tube we consult the graph labeled Figure 2 and find that the drag form factor (C_DA) is 0.33 $in.^2$

3. The next step is to calculate the **ballistic coefficien**t of the rocket during powered (thrusting) flight. The average weight of the rocket (includes weight of rocket and engine) is used in this calculation. The average weight is the initial weight (W_I) minus one-half of the propellant weight (1/2 $W\rho$). Refer to Figure 5A for the necessary data on A8-3 engines. The equation for calculating the **ballistic coefficient** during thrusting (Thrusting β) is

$$\text{Thrusting } \beta = \frac{W_I - \tfrac{1}{2} Wp}{C_DA}$$

$$= \frac{1.00 \text{ oz.} - 0.055 \text{ oz.}}{0.33 \text{ inch}^2}$$

$$= \frac{0.945 \text{ oz.}}{0.33 \text{ inch}^2}$$

$$= 2.86 \text{ ounces per square inch}$$

This quantity, the **thrusting ballistic coefficient,** gives you an idea of the amount of mass per square inch of frontal area. This should help you to better understand how well the rocket can travel during thrusting. <u>The higher the **ballistic coefficient** during thrusting, the faster the rocket can travel for a given amount of thrust.</u>

4. Once the **thrusting ballistic coefficient** has been determined, refer to the graphs for the A8 engine to determine the **burnout velocity** and **burnout altitude** for this rocket.

S_B = **burnout altitude** from Figure 5A = 61 ft.

V_B = **burnout velocity** from Figure 5B = 295 ft. per second

5. Once the propellant is gone, calculations for the coasting phase of the flight should use the **coasting ballistic coefficient** which is calculated using the weight of the rocket vehicle plus the weight of the engine without propellant. The equation for calculating the **ballistic coefficient** during coasting (Coasting β) is

$$\text{Coasting } \beta = \frac{W_I \; Wp}{C_D A}$$

$$= \frac{1.00 \text{ oz.} - 0.110 \text{ oz.}}{0.33 \text{ inch}^2}$$

$$= \frac{0.89 \text{oz.}}{0.33 \text{ inch}^2}$$

$$= 2.70 \text{ ounces per square inch}$$

The **coasting ballistic coefficient** gives you an idea of the amount of mass per square inch of frontal area during coasting. The higher the **coasting ballistic coefficient,** the greater the distance the rocket can coast for a given velocity. A low **coasting ballistic coefficient** indicates a rocket with relatively high drag.

6. The distance which the rocket will coast upward is read from the **Coasting Altitude** graph.

Coasting altitude from Figure 11A = 510 feet

7. Determine the total altitude the rocket can reach by adding together the **burnout altitude** and the **coasting altitude.**

S_B = **Burnout altitude**	=	61 ft.
S_C = **Coasting altitude**	=	510 ft.
		571 ft.

8. The time that the rocket will coast is determined from the **Coasting Time** graph.

t_c = Coasting time from Figure 11B = 4.9 seconds

Since we selected an A8-3 engine, the rocket will not reach the maximum possible altitude before parachute ejection. The A8-5 engine (no longer available) should be used rather than the A8-3 engine if maximum altitude is to be reached.

Five problems are explained and solved on pages 26-34. To gain skill in using these graphs, you can solve a problem, then check your work against the complete solution for that problem.

ACCURACY OF THIS METHOD FOR CALCULATING ALTITUDES

Does a question arise in your mind about the accuracy of this method? Can something this easy be correct? Those of you who have the June, 1964 issue of Model Rocket News can check that the altitude found for a rocket without considering drag was 2600 feet while the same rocket launched with the old A8-4 engine (no longer available) was 750 feet. This indicates that the altitude calculations which ignored the effects of drag had an error of over 300% as compared to altitude calculations which included drag.

EFFECTS OF STREAMLINING

Now let's take a quick look at some of the effects "streamlining" can have for the same rocket. Suppose we have two people build this same rocket. Bill's is unpainted, unsanded and the fins are square. Carl's bird, on the other hand, has a beautiful smooth paint job, is fully waxed and the fins have a nice airfoil shape. For comparison purposes let's assume Bill's **aerodynamic drag coefficient** has increased 20% over the C_D of 0.75 we previously used (Bill's C_D now equals 0.9) and Carl's **drag coefficient** has been reduced by 20% (Carl's C_D now equals 0.6).

Going through the same steps of the sample problem, you will find Bill's rocket will reach 530 feet while Carl's will reach 621 feet.

"STREAMLINING" - ITS EFFECT ON DRAG

	Bill	Carl
Drag Coefficient (C_D)	.9	.6
Thrusting Ballistic Coefficient (β_t)	2.42 oz./in.^{2}	3.63 oz./in.^{2}
Burnout Altitude (S_B)	60 ft.	61 ft.
Burnout Velocity (V_B)	280 ft./sec.	285 ft./sec.
Coasting Ballistic Coefficient (β_C)	2.28 oz./in.^{2}	3.42 oz./in.^{2}
Coasting Altitude (S_C)	470 ft.	560 ft.
Coast Time (t_c)	4.6 sec.	5.3 sec.
Total Altitude (S_B+S_C)	530 ft.	621 ft.

As one becomes familiar with the method in this report you can find many other interesting aspects of aerodynamics to consider. It is very easy to investigate the effect on performance of any of the variables by just computing altitudes for various engine types, heavier and lighter rocket weights and different values of drag. With this kind of approach, one soon gains a real understanding of these previously abstract principles.

INSTRUCTIONS FOR USING THE GRAPHS ON THE FOLLOWING PAGES

The basic steps in calculating the altitude that will be reached by a specific model rocket powered by a specific type of model rocket engine are as follows:

1. Gather data on a specific example -
a) Find frontal area of model rocket (body tube size).

A few simple calculations using only arithmetic operations (adding, subtracting, multiplying and dividing) plus use of the graphs on the following pages will enable you to calculate with reasonable accuracy the altitudes to which your model rockets will fly with different engines.

b) Determine weight of the model rocket without engine (from catalog or by actually weighing the model).
c) Secure weight of the engine and weight of the propellant (from catalog).

2. Determine the drag form factor (C_DA). This can be done for model rockets made with standard Estes body tubes by reference to Figure 2.

3. Calculate the **ballistic coefficient** (β_t) of the model rocket during thrusting -

$$\text{Thrusting} \quad \beta = \frac{W_I \; 1/2 Wp}{C_DA}$$

4. Read the **burnout altitude** (S_B) and the **burnout velocity** (V_B) from the appropriate graphs.

5. Calculate the new **ballistic coefficient** (βc) for model rocket during coasting. (All of the propellant is gone) -

$$\text{Coasting} \quad \beta = \frac{W_I \; Wp}{C_DA}$$

6. Read **coasting altitude** from the appropriate graph.

7. Determine altitude your model rocket will reach by adding the **coasting altitude** to the **burnout altitude**.

8. Determine the **coasting time** from the appropriate graph and check to be sure that you selected an engine with the proper delay.

CLUSTERED ROCKETS APPLICATION

The charts can also be used for any identical type motors clustered in a stage by making the following modifications.

During thrusting use the V_B and S_B graphs with

$$W_I = \frac{\text{Actual Weight}}{\text{Number of motors in cluster}} = \frac{W_{actual}}{N}$$

and:

$$\beta T = \frac{W_{actual} - N\,(1/2 W_P)}{C_DA}$$

During coasting use:

$$\beta c = \frac{W_{actual} - N(W_P)}{C_DA}$$

Nothing else is required.

As an example, let's look at the Gemini-Titan GT-3 (no longer available) powered by two B6-4 engines. Its body is a BT-70 tube, and let's again assume C_D = .75. The rocket weighs 3.8 ounces.

Now:

$$W_I = \frac{W_{actual}}{\text{Number of Motors}}$$

$$= \frac{3.8 \text{ oz.} + 2(0.78 \text{ oz})}{2}$$

$$= 2.68 \text{ oz.}$$

$$C_DA = 2.85 \text{ in.}^2$$

$$\beta_t = \frac{W_{actual} - N(1/2 W_P)}{C_DA}$$

$$= \frac{5.36 \text{ oz.} - 2\left(\frac{0.22 \text{ oz.}}{2}\right)}{2.85 \text{ in.}^2}$$

$$= 1.80 \frac{\text{oz.}}{\text{in.}^2}$$

S_B from figure 7A = 84 ft.

V_B from figure 7B = 175 ft./sec.

$$\beta_c = \frac{W_{actual} - N(WP)}{C_DA}$$

$$= \frac{5.36 \text{ oz.} - 2(0.22 \text{ oz.})}{2.95 \text{ in.}^2}$$

$$= 1.67 \frac{\text{oz.}}{\text{in.}^2}$$

S_C from figure 11A = 230 ft.

Apogee point = $S_B + S_C$
= 84 ft/ + 235 ft.
= 319 ft.
t_c = 3.3 sec.

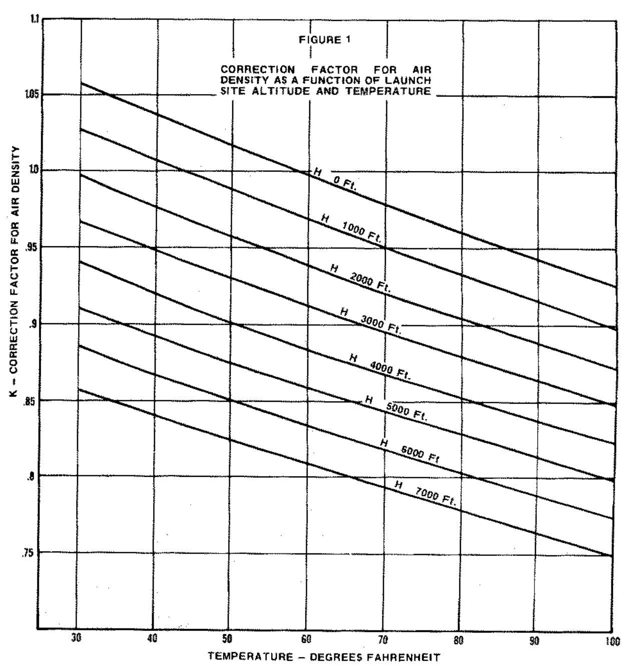
Note that our choice of a 4-second delay will pop the parachute close to the apogee.

THE EFFECT OF LAUNCH ALTITUDE AND LAUNCH TEMPERATURE VARIATIONS

Our basic drag equation D = C_DA 1/2 ρV2 shows that the drag force (D) is among other things proportional to air density (ρ). Density of the air is both a function of altitude and temperature. Figure 1 presents a correction factor (K) for density that is based on the tabular data presented on page 92 or G.H. Stine's Handbook (reference 4). This factor (K) can fortunately be included in the ballistic coefficient (β) as follows:

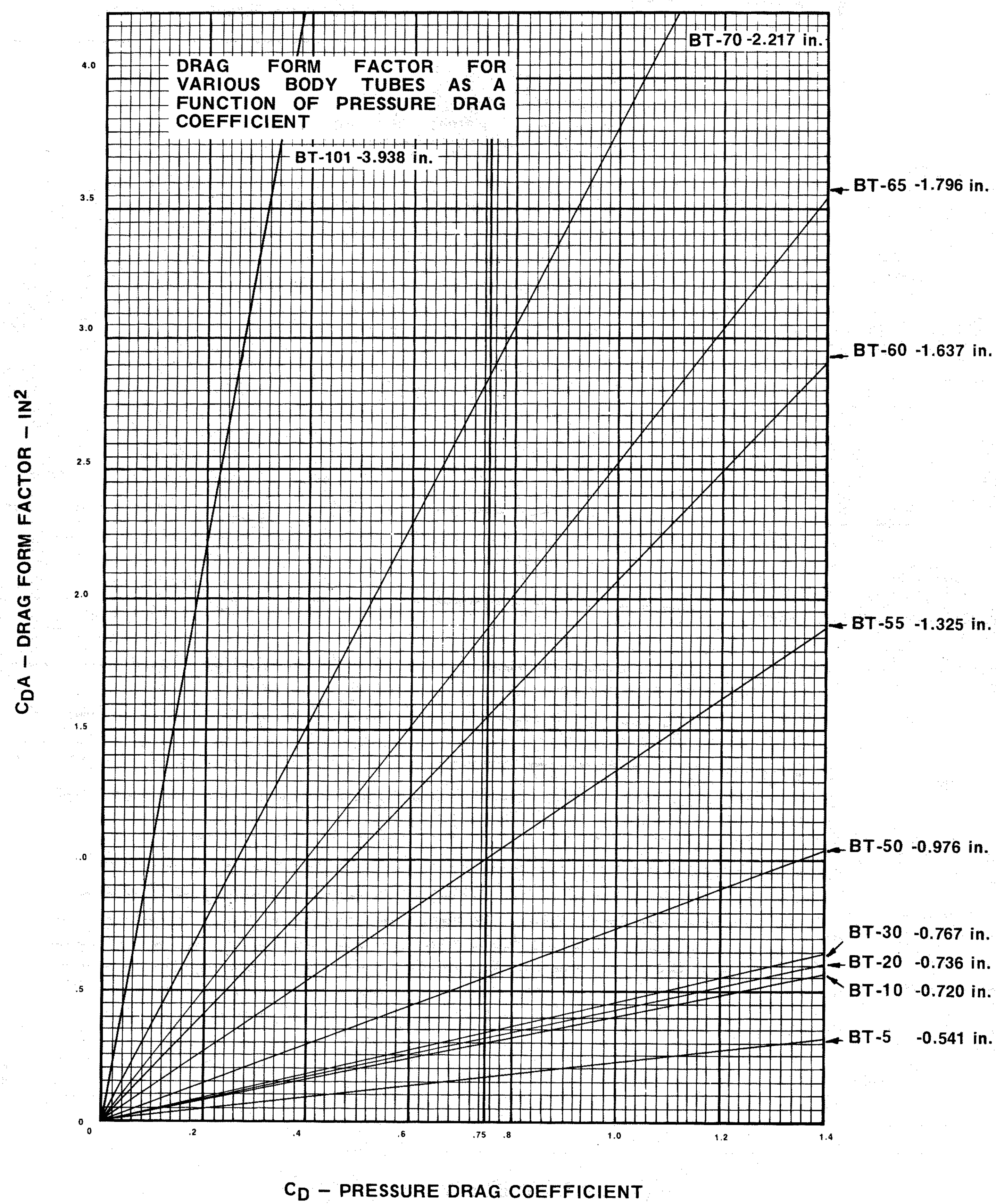
$$\beta = \frac{W}{C_DA\,K}$$

Altitude calculations are performed just as described before with the exception of this single modification. The reason it can be included at right in the ballistic coefficient will become apparent to those who follow the derivation of the motion equations.



Drag Form Factors

FIGURE 2



1/4A3

1/4 A3

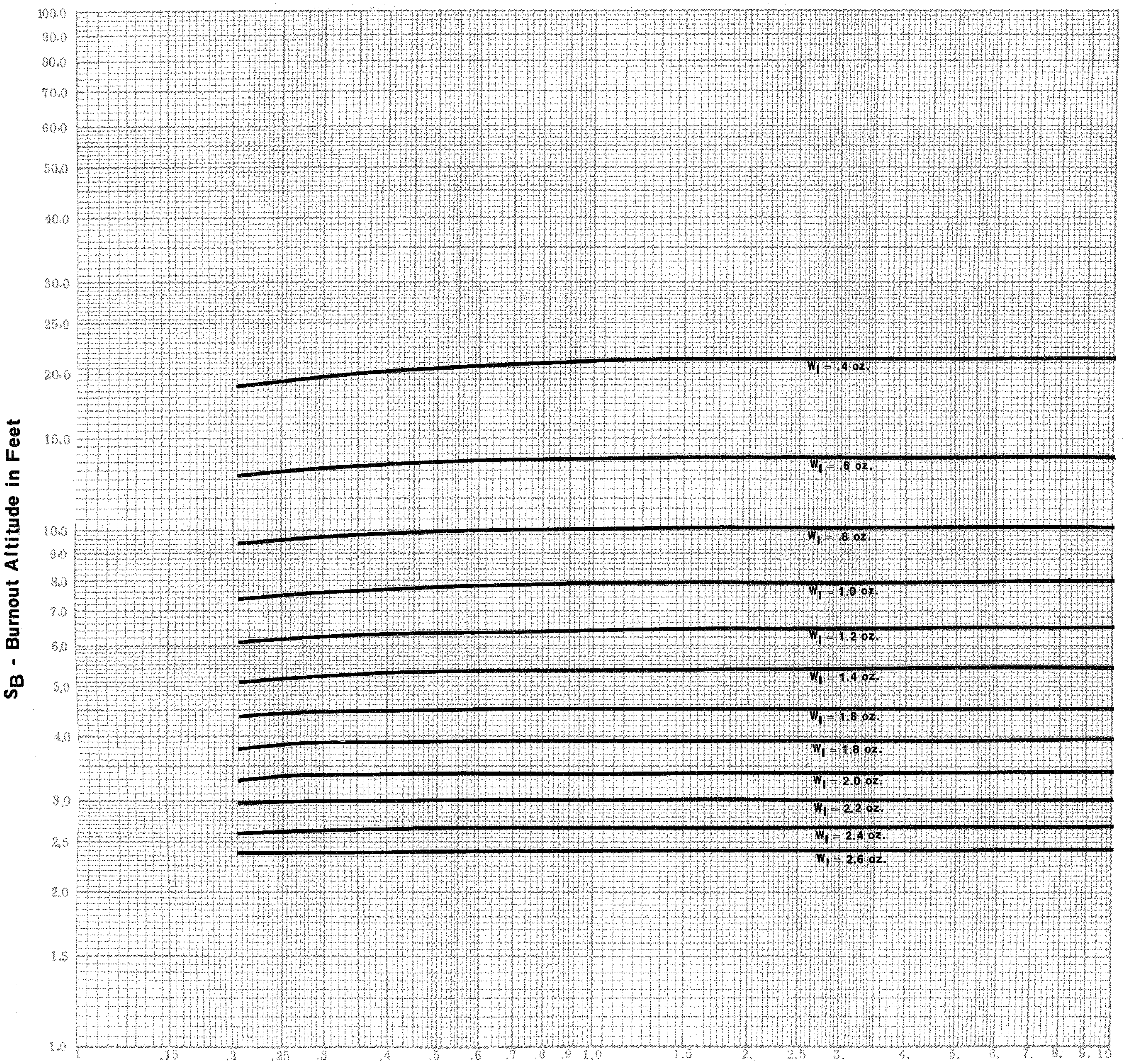
Burn Time t_b = .24 Sec.

Propellent Weight W_P = .0270 Oz.

$1/2\ W_P$ = .0135 Oz.

Average Thrust T = 9 Oz.

FIGURE 2A
Burnout Altitude (S_B) as a function of Initial Weight (W_I) and Ballistic Coefficient (β_t).

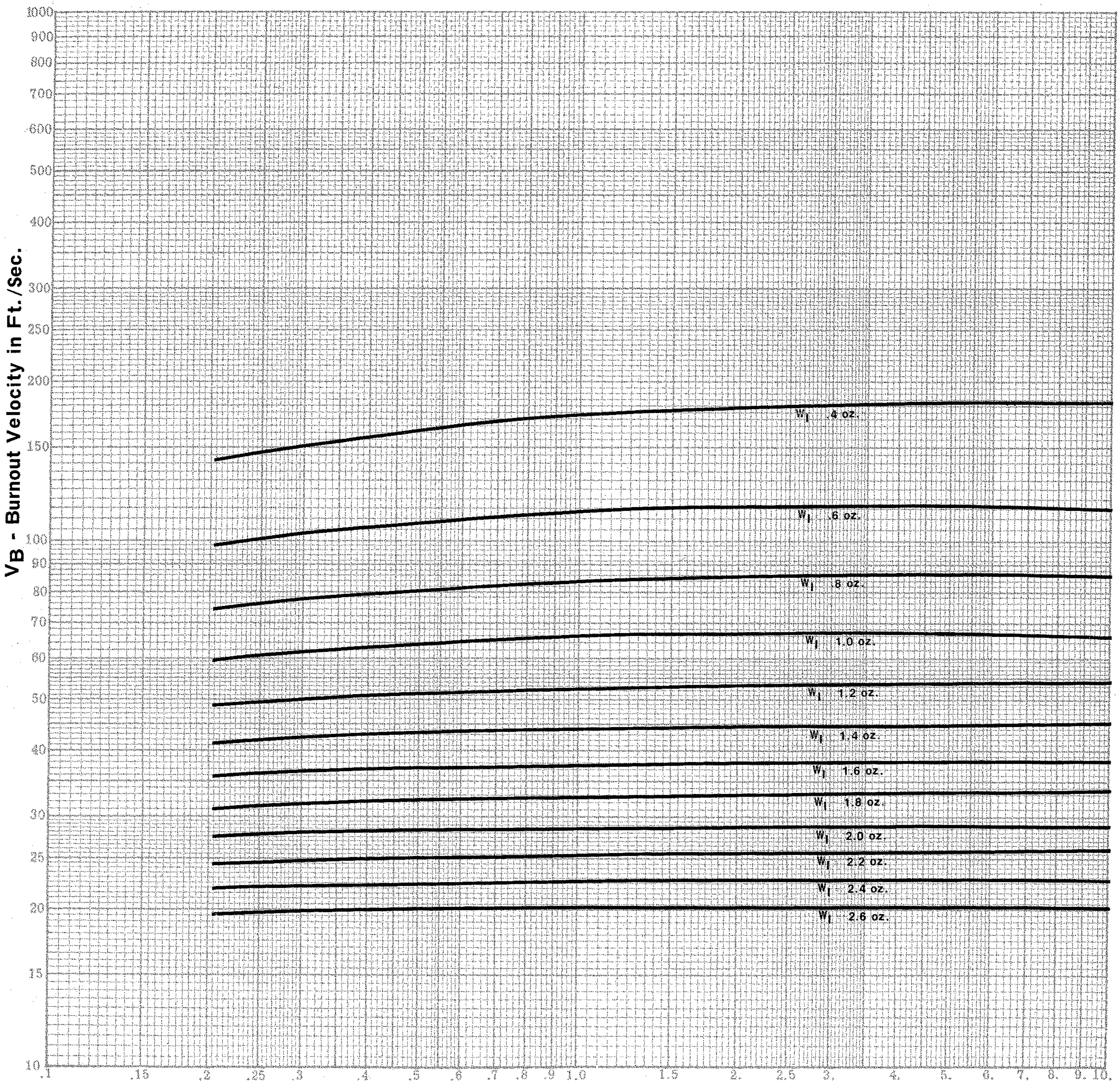


β_t = Ballistic Coefficient $= \frac{W}{C_D A} = \frac{\text{ounces}}{\text{inch}^2}$

1/4A3

1/4 A3
Burn Time t_b = .24 Sec.
Propellent Weight W_P = .0270 Oz.
1/2 W_P = .0135 Oz.
Average Thrust T = 9 Oz.

FIGURE 2B
Burnout Velocity (V_B) as a function of Initial Weight (W_I) and Ballistic Coefficient (β_t).



β_t = Ballistic Coefficient = $\frac{W}{C_D A} = \frac{\text{ounces}}{\text{inch}^2}$